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SUMMARY OF FLUTTER EXPERIENCES AS A GUIDE TO THE PRELIMINARY
DESIGN OF LIFTING SURFACES ON MISSILES

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SUMMARY OF FLUTTER EXPERIENCES AS A GUIDE TO THE PRELIMINARY DESIGN OF LIFTING SURFACES ON MISSILES¹

By Dennis J. Martin

The purpose of this paper is to present a limited review of some experiences in flight testing of missiles and of wing flutter investigations that may be of interest in missile design. Several types of flutter which may be of concern in missile studies are briefly described. Crude criteria are presented for two of the most common types of flutter to permit a rapid estimate to be made of the probability of the occurrence of flutter. Many of the details of the flutter picture have been omitted, and only the broader elements have been retained so as to give the designer an over-all view of the subject.

There are many different types of flutter that may be encountered on airplanes, propellers, helicopters, missiles, and so forth, and there are many speed ranges and conditions in which the phenomena are widely different. Broadly speaking, the phenomenon of flutter is generally concerned with vibrations or oscillations of a lifting surface. Oscillations of a lifting surface give rise to oscillations of the aerodynamic forces which in turn, under certain conditions, may reinforce or increase the oscillations to dangerous amplitudes. Some types of flutter may be mild; others may be disastrous. Flutter may involve fully established flow or broken-down flow, high or low frequencies of the structure, and one or more modes of vibration. Examples of some of the more common modes that may interact during flutter are given in figure 1.

The first example shown is the most common type of flutter encountered in which the elastic modes of the wing, wing bending and wing twisting or torsion, combine to extract energy from the air stream, that is, to produce flutter. In addition, a control surface may interact significantly with these motions to produce other types of flutter.

The second example shown in figure 1 illustrates a type of flutter which involves only one motion or degree of freedom. The type of flutter illustrated occurs at high angles of attack and is commonly known as stall flutter. Only a torsional twisting motion of the wing is present. There are other motions that may produce a single-degree flutter of the type illustrated by this stall-flutter case. Examples are: aileron buzz, single-degree bending oscillations of swept wings, and single-degree pitching oscillations of a wing.

The third example in figure 1 illustrates a type of flutter in which the motion of the whole fuselage enters significantly into the

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flutter. This example illustrates a pitching motion of the entire missile combined with a bending motion of the wing. Other body motions (rolling, yawing, vertical translation, and so forth) also may enter into flutter.

The missile not only experiences many of the flutter problems encountered with aircraft but also presents many new and different problems, depending upon the design and purpose of the missile. Examples are: skin flutter, flutter of automatic controls or servomechanisms, and flutter of short wings with ram jets or external stores.

Many of these types of flutter can best be studied by difficult experiments; others require long and tedious theoretical investigations. For the more common types there exist sufficient experimental data to evaluate simple criteria. In general so many factors enter into a flutter case that a comprehensive criterion becomes quite unwieldy. Simple criteria must neglect or restrict many parameters. Furthermore, there are possibilities for exception; hence any simple criterion should not be considered as perfectly general. In spite of these limitations a criterion does have some usefulness in estimating the probability of a particular type of flutter occurring for a given configuration.

In this paper two simple criteria are presented. The first is for the most common type, the wing bending-torsion flutter illustrated by the first example of figure 1. Another is presented for stall flutter shown by the second example, and a brief discussion is included of pitch-bending flutter which is illustrated by the third example.

In order to illustrate the significance of the first criterion to be presented for the most common type, wing bending-torsion flutter, figure 2 (see references 1 and 2) has been prepared to show the flutter behavior of wings over a range of Mach numbers.

Shown in this figure is the actual flutter Mach number plotted against a calculated quantity N which is dependent upon the wing stiffness, center-of-gravity location, mass ratio, certain aerodynamic quantities, and so forth. The parameter N is an orderly combination of many of the parameters that are important in flutter. The flutter expert may recognize this N as the calculated, two-dimensional incompressible flutter speed divided by the velocity of sound. Typical curves for wings of zero sweep and full-span aspect ratios of 2 and 7 are shown and an approximate curve is shown for a wing of 60° sweep of aspect ratio 4. For the aspect-ratio-7 wings it can be seen that, as the value of N is increased, for example, by increasing the wing torsional stiffness or by increasing the operating altitude, the flutter Mach number increases and a value of N is eventually reached which will not produce an intersection with the flutter boundary. The significance of this result is that, if the wing is stiff enough or if

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the quantity N is large enough so that flutter is not encountered in this region around a Mach number of 1 for the higher aspect ratios, flutter of the type considered herein would not be expected to occur at higher Mach numbers.

It is this maximum or critical value of the quantity N that is of interest in missile design, because missiles must operate throughout the Mach number range. It must be emphasized that the curves for other aspect ratios, sweep angles, and so forth may appear quite different from the one shown for the aspect-ratio-7 wings and hence may have a different critical Mach number range and a different value of N required for the wing to be flutter-free.

If all the critical values of this parameter N that are necessary to avoid flutter were known for the various aspect ratios, sweep angles, thickness ratios, section properties, and so forth, the flutter problem for missile design would be greatly simplified.

An investigation is now under way in the Langley supersonic flutter apparatus and is aimed at defining values of N for various supersonic Mach numbers. Many of these unpublished data, together with many data from the Langley Pilotless Aircraft Research Division on missiles and bombs which experienced no known flutter difficulties as well as on missiles on which flutter was attained, have been accumulated. This store of experience has been compared to a simplified and modified criterion which groups the significant parameters in a manner similar to that used for N in an attempt to establish limits of the critical values of the structural and aerodynamic requirements for a wing to be flutter-free. This criterion is based on modifications to an approximate flutter formula proposed by Theodorsen and Garrick (reference 3). This formula was for high-aspect-ratio, heavy wings having a low ratio of bending to torsional frequency. The application of modifications of this formula to include low-aspect-ratio wings including swept and highly tapered wings is certainly stretching the basic formula; however, in the present study this approach has been made and the parameters have been juggled until there seems to be a reasonable coherence in the results.

For simplicity this modified criterion is broken down into simple geometric dimensions and structural properties. (This modification is described in the appendix.) The experimental data which have been accumulated are then compared and an attempt is made to bracket the safe wings and to bracket the unsafe wings. This comparison is made in figure 3.

Plotted against the effective shear modulus of the wing material is p/p_0 , the fluid pressure compared to standard pressure, times $\frac{\lambda + 1}{2}$,

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where λ is the taper ratio, times a quantity X which is obtained from the geometric dimensions of one wing panel. The abscissa of this figure G_E is the effective shear modulus of the wing structure and is indicated for wings of solid wood, magnesium, aluminum, titanium, and steel. A solid wing of, say, aluminum would fall at the point marked A along the abscissa while a fabricated wing of aluminum would have a lower effective G_E and would fall somewhat to the left, depending upon the skin thickness and spar size. The value of G_E can be determined for fabricated wings from a measured value of the torsional stiffness parameter JG by the relation $G_E = \frac{6(JG)_{\text{measured}}}{ct^3}$, where c is the chord and t is the thickness. The quantity X noted in the ordinate is shown in figure 4 as a function of the panel aspect ratio for constant values of the thickness ratio in the streamwise direction. It must be remembered that the abscissa of this figure is the exposed aspect ratio of only one wing panel as distinguished from the normal aspect ratio which includes both wings and the fuselage.

Shown in figure 3 are data taken from subsonic, transonic, and supersonic wind tunnels and from rocket and bomb-drop tests for both swept and unswept wings. The open points are for missile wings that traversed the Mach number range to at least a Mach number of 1.3 or higher without known failure. The solid points are for missile and wind-tunnel tests where flutter or failure occurred. It must be pointed out that some of the data are for missiles that were designed primarily for aerodynamic research. The instrumentation of these missiles was, therefore, not usually of a type that could definitely indicate that no oscillations occurred for the cases represented by open points or that the failures for the cases represented by the solid points were due to flutter rather than some other cause.

The many data shown tend to indicate that two regions can be defined in which the open and solid points are reasonably well separated and the flutter region is established. The shaded area indicates a probable division based on the existing data. This chart is useful in estimating the probability of the occurrence of flutter of the bending-torsion type for a given configuration. A designer may see where his design lies with respect to many other designs which did or did not experience flutter troubles.

As an illustration, suppose a design had an exposed-wing-panel aspect ratio of 2, a streamwise thickness ratio of 4 percent; from figure 4 a value of X of about 1.25×10^6 pounds per square inch is indicated. If the missile were ground-launched, that is, at standard pressure $\frac{P}{P_0} = 1$, and if the wing were untapered, that is $\frac{\lambda + 1}{2} = 1$, then the ordinate of figure 3 for this design would be 1.25×10^6 , and

if the construction were solid magnesium it would plot in the flutter region and would most probably be unsafe. If made of solid aluminum, it would be marginal. Further detailed analysis or experiment would be needed to complete this design. However, if the wing were solid steel, it would probably be safe, at least insofar as the bending-torsion type of flutter considered is concerned.

As another example, suppose the material of construction has already been selected, titanium for example. In order to allow a reasonable margin of safety an ordinate of figure 3 of about 0.8 or less might be specified. If the wing were untapered and ground-launched, the ordinate of figure 4 is then 0.8 and it can be seen that for a panel aspect ratio of 1 a thickness ratio of 2.5 percent is required. For an aspect ratio of 2 a thickness of 4.5 percent is required, and for an aspect ratio of 3 a thickness of about 6.5 percent must be used so that the design may most likely be free of the bending-torsion type of flutter for which this figure applies.

There are, however, many other types of flutter that may occur under certain conditions and they must also be investigated. With a change in the type of flutter, a change must be made in the type of criterion. As mentioned previously the type of flutter that may be encountered depends upon the design and purpose of the missile. As an example of the dependence of the type of flutter upon the use of the missile, it may be mentioned that high-angle-of-attack flutter, that is, stall flutter, would probably be considered as possible only for missiles that are required to maneuver sharply. This conclusion is quite true for ground-launched missiles; however, any air-launched missile that is carried externally may be subject to large angles of attack during airplane maneuvers prior to launching and thus may become subject to stall flutter.

During a recent bomb-drop test at Langley, a missile-wing failure occurred while the bomb was attached to the airplane. The failure occurred at speeds considerably below the flutter speed which was later obtained with an identical wing that was protected from the air stream while attached beneath the airplane. An investigation of stall flutter of thin wings was begun. Although missile data on stall flutter are not readily available, a brief discussion of the stall flutter of thin wings and stall flutter of propellers may serve as a rough guide for missile design.

Figure 5 illustrates the flutter behavior of a typical wing at low speeds as the wing produces lift. The ordinate is a nondimensional flutter-speed coefficient, V , the flutter speed, divided by b , the half-chord, and ω_α , the torsional frequency. The abscissa is the angle of attack. As the angle of attack is increased the flutter speed is

reduced drastically. The flutter speed falls rapidly and a minimum is reached at an angle of attack near the stalling angle of the wing.

The flutter encountered at the low angles of attack is of the bending-torsion type which was discussed in the previous figures and was seen to be strongly dependent upon the material of construction. At high angles of attack the flutter occurs essentially in only a torsional mode and this minimum value of the flutter-speed coefficient $V/b\omega_\alpha$ has been found to be nearly equal to 1 for almost all wings and propeller blades at low speeds, independent of the material of construction. These results have been confirmed for both wings and propeller blades, and the results are thought to be generally valid for the subcritical-flow speed range. Now, this curve is typical for low-subsonic-speed data. It has not been determined what these curves might look like at supersonic speeds; however, a study of the minimum value of $V/b\omega_\alpha$ as affected by Mach number has indicated a beneficial effect at higher speeds. The experimental work of Baker at Langley (reference 4) has suggested that the quantity $b\omega_\alpha$ referred to the speed of sound was a significant parameter for determination of configurations that would be free of stall flutter.

Figure 6 has been prepared to show a comparison of experiment with this parameter $b\omega_\alpha/a$ for a range of Mach numbers. Shown in this figure is the flutter Mach number plotted against $b\omega_\alpha/a$. The curve shown represents the boundary where stall flutter could begin for a given value of $b\omega_\alpha/a$. As $b\omega_\alpha/a$ is increased, for example, by increasing the chord or increasing the torsional frequency, a value is noted to be reached which will not produce an intersection with the flutter boundary. The result is quite similar to the situation that occurred for the bending-torsion flutter (fig. 2).

Baker has shown that a value of $b\omega_\alpha/a$ of at least 0.5 is required for a propeller to be completely free of stall flutter. A. G. Rainey (reference 5) at Langley has substantiated this value of 0.5 for unswept wings of moderate aspect ratio and low structural damping but has indicated, however, that aspect ratio, structural damping, and sweepback may have an influence on the critical value of $b\omega_\alpha/a$. These effects are not well-determined and cannot at present be included in a design chart. The value of $b\omega_\alpha/a$ of 0.5 has nevertheless been used to prepare a design chart for solid unswept wings. Since the torsional frequency times the chord for solid unswept wings is a function only of the length-to-chord ratio and the thickness ratio and is essentially independent of the material (that is, for such common materials of construction such as steel, aluminum, and magnesium), the design chart (fig. 7) is presented in terms of the length-to-chord ratio and the thickness ratio required to attain a value of $b\omega_\alpha/a$ of 0.5. A speed of sound of 1100 feet per second is assumed.

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Wings having geometric quantities which plot above the solid line of figure 7 may experience stall flutter if sufficiently high angles of attack are encountered. It must be remembered that the boundary indicated is for the most critical condition. If the speed range is traversed at low angles of attack a design may be well above the boundary of this figure without encountering difficulties. This boundary represents only the conditions required for the wing to be completely free of stall flutter throughout the speed range at any angle of attack. The margins of safety for this criterion are not established, and the criterion may have to be modified as more information and data are made available on sweepback, aspect ratio, and structural damping.

The significance of free-body modes in flutter has been of interest for some time. The problem was considered in early British work on flutter involving the mobility of the fuselage. Recently Broadbent (reference 6) and Houbolt (reference 7) have discussed this problem and have developed simple criteria based on the position of the nodal line. The type of flutter involving missile pitching and wing bending is dependent upon the moment of inertia in pitch of the missile and the bending stiffness of the wing as well as the wing location with respect to the center of gravity of the missile. For wings which meet the torsional-stiffness criterion for the bending-torsion type of flutter but, however, are weak in bending, this type of flutter may become important for some wing locations if the missile has a high moment of inertia in pitch. In some flutter tests by the use of rocket vehicles at Langley, several failures have occurred and seemed to involve principally wing bending and missile pitching. The frequency of flutter was somewhat below the first bending frequency of the wing, near the short-period oscillation of the body. Analyses have been made (reference 8), and the effect of wing location is illustrated in figure 8.

The ordinate in figure 8 is the flutter-speed coefficient $V/b\omega_h$; in this case the first bending frequency ω_h is used. The abscissa is the nondimensional distance of the wing behind the center of gravity of the body. The dashed line represents the conventional bending-torsion type of flutter while the solid line shows the effect of inclusion of a body degree of freedom. This flutter speed is much lower than for the bending-torsion type for rearward locations and much higher for forward locations. The significant conclusion that can be drawn from these studies of pitch-bending flutter is that the most important consideration is the inclusion of the proper degrees of freedom or modes in the analysis. Moreover, the observation can be made that, with the change in the type of flutter, a change occurs in the type of flutter criterion; thus in this case the critical speed is affected strongly by the bending stiffness and not by the torsional stiffness as for the case of bending-torsion flutter.

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This paper has presented material that may be applicable to spot design. The proximity of a design to common types of flutter may be indicated; however, the margins of safety can only be determined by more precise analysis and by controlled experiments.

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APPENDIX

An empirical expression for flutter speed as given by Theodorsen and Garrick (reference 3) for heavy, high-aspect-ratio wings with a low ratio of bending to torsional frequency is given by

$$\frac{V_f}{b\omega_\alpha} = \sqrt{\frac{r_\alpha^2}{\kappa} \frac{1/2}{\frac{1}{2} + a + x_\alpha}}$$

It has been found that V_f is better represented if the quantities are determined at the three-quarter spanwise station. If ϵ is substituted for $\frac{\frac{1}{2} + a + x_\alpha}{2}$, the distance of the center of gravity of the section behind the quarter-chord position, this expression becomes, upon squaring,

$$V_f^2 = \frac{c_{0.75}^2 \omega_\alpha^2 r_{\alpha 0.75}^2}{16\kappa\epsilon}$$

Coleman (reference 9) has given the frequency of a beam of constant thickness ratio and tapered linearly in chord in terms of an untapered uniform beam having the same root chord

$$(\omega_\alpha)_{\text{tapered}} = (\omega_\alpha)_{\text{untapered}} f_1(\lambda)$$

where f_1 may be approximated by $1 + 1.87(1 - \lambda)^{1.6}$.

If geometrically similar sections are assumed at all spanwise stations, the expressions for ω_α , $r_{\alpha 0.75}$, and κ may be substituted into the expression for V_f^2 and the equation becomes

$$\frac{V_f^2}{a^2} = \frac{\pi c_{0.75}^2 f_1^2(JG)_0}{4\epsilon \rho_a c_0^4 L^2 a^2}$$

where a is the velocity of sound. For a solid, thin airfoil J can be closely approximated by

$$J \approx \frac{ct^3}{6}$$

For a fabricated section J is extremely difficult to calculate. The value of JG of a section may be experimentally measured and an equivalent solid section may be assumed, that is,

$$(JG)_{\text{measured}} = J_S G_E$$

where G_E is the effective shear modulus of the equivalent section, that is,

$$G_E = \frac{6JG}{ct^3}$$

Since t/c is the thickness ratio and $L/c_{0.50}$ is the panel aspect ratio, the expression for V_f^2/a^2 may be reduced to the following form by using the relation $c_{0.75} = c_{0.50} f_2(\lambda)$ where $f_2 = \frac{1 + 3\lambda}{2 + 2\lambda}$:

$$\left(\frac{V_f}{a}\right)^2 = \frac{\pi f_1^2 f_2^2 \left(\frac{t}{c}\right)^3 G_E}{24 \epsilon \rho_a A^2 a^2}$$

An aspect-ratio correction of the form $\frac{A}{A + n}$ has frequently been used.

Reference 10 has suggested a value of $n = 2$. This value of $n = 2$ has been used with success in flutter criteria in reference 11. In this investigation values of $n = 1, 2$, and 3 were tried and the value of $n = 2$ gave the more consistent results. If the value of $n = 2$ is used, the expression for $(V_f/a)^2$ becomes

$$\left(\frac{V_f}{a}\right)^2 = \frac{G_E}{\frac{24 \epsilon}{\pi} \rho_a a^2 \frac{A^3}{\left(\frac{t}{c}\right)^3 (A + 2)} \frac{1}{f_1^2 f_2^2}}$$

The expression for $1/f_1^2 f_2^2$ is closely approximated by $\frac{\lambda + 1}{2}$, and $\rho_a a^2$ can be rewritten in terms of ρ_0 , a_0^2 , and p/p_0 . The results apply to air as a medium. If a different gas is used, the relative specific-heat ratio must also be included. A value of $\epsilon = 0.25$ will be assumed; however, for sections whose center of gravity is far from the 50-percent-chord position a correction may be required. Since the shear modulus G_E is usually expressed in pounds per square inch,

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$\rho_0 a_0^2$ must also be expressed in pounds per square inch. The expression for $(V_f/a)^2$ then becomes

$$\left(\frac{V_f}{a}\right)^2 = \frac{G_E}{39.3 \frac{A^3}{\left(\frac{t}{c}\right)^3 (A+2)} \left(\frac{\lambda+1}{2}\right) \frac{p}{p_0}}$$

Thus, if a critical value of $(V_f/a)^2$ exists, a plot of the denominator of this expression for various materials or values of G_E for missiles and wind-tunnel tests may permit the safe wings to be separated from the unsafe wings. The expression

$$X = \frac{39.3 A^3}{\left(\frac{t}{c}\right)^3 (A+2)}$$

is calculated and plotted in figure 4.

The value of X can be determined from figure 4 from the thickness ratio and aspect ratio of one exposed wing panel. This value of X is multiplied by $\frac{\lambda+1}{2}$ and by p/p_0 to obtain the ordinate of figure 3.

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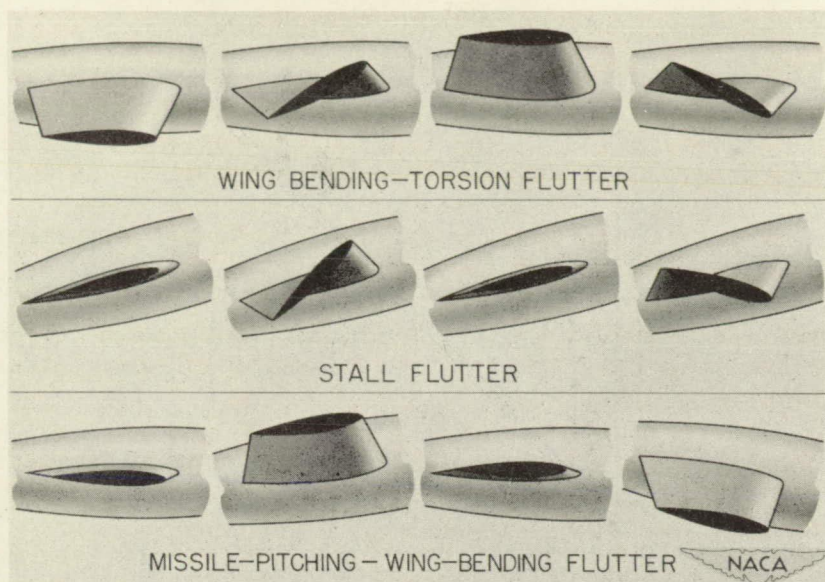


Figure 1.- Examples of flutter modes.

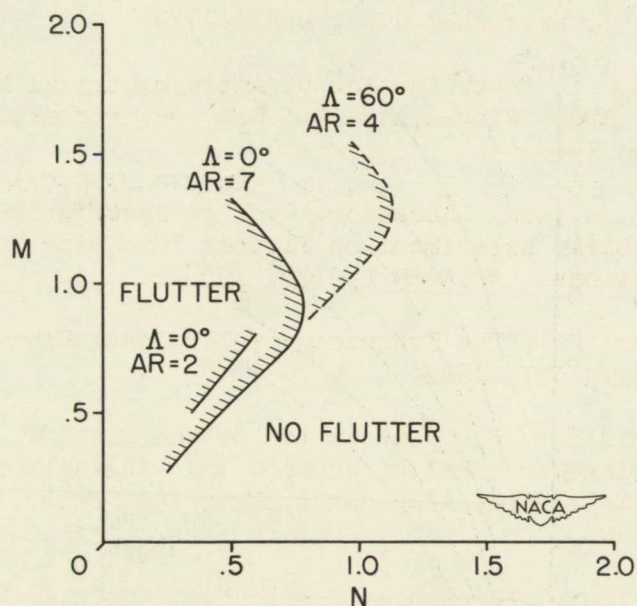


Figure 2.- Trend study of swept and unswept wings at transonic speeds.

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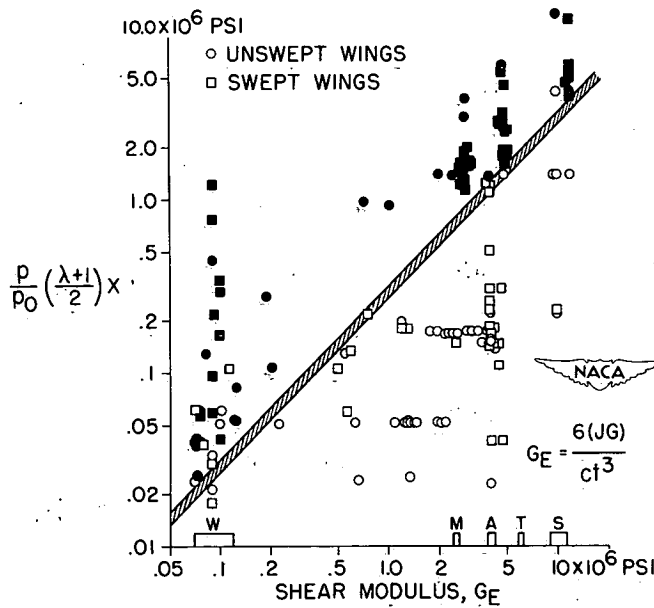


Figure 3.- Composite chart for bending-torsion flutter.

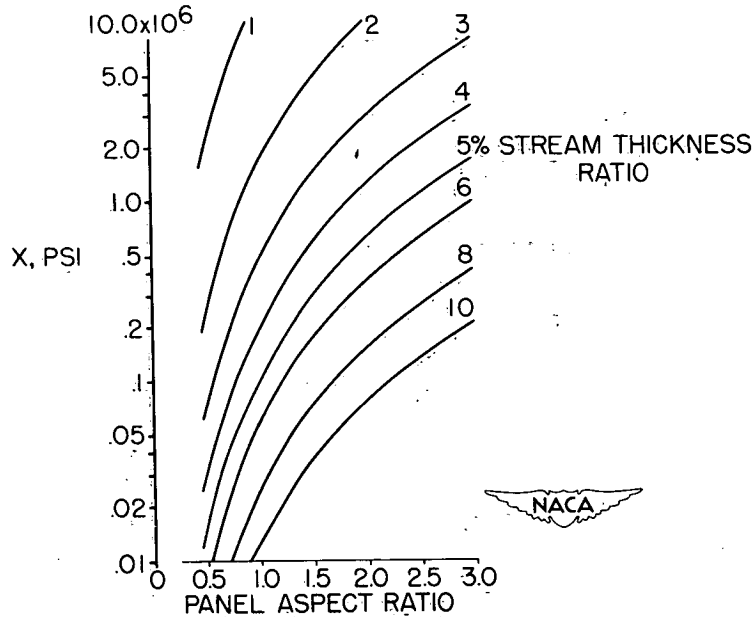


Figure 4.- The parameter X as a function of panel aspect ratio and thickness ratio.

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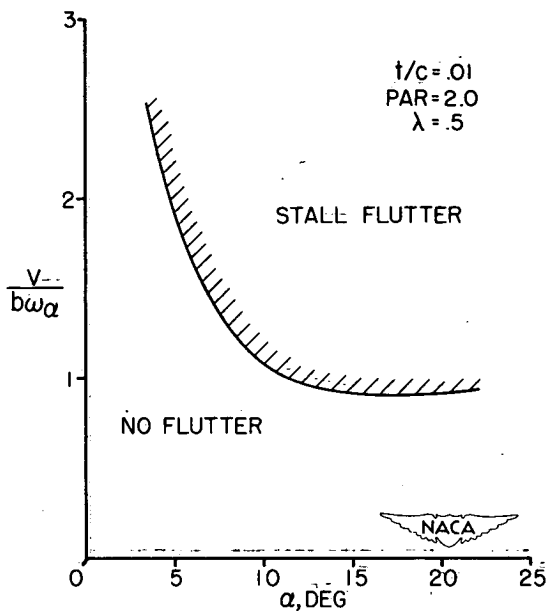


Figure 5.- Flutter behavior of a typical wing at angles of attack.

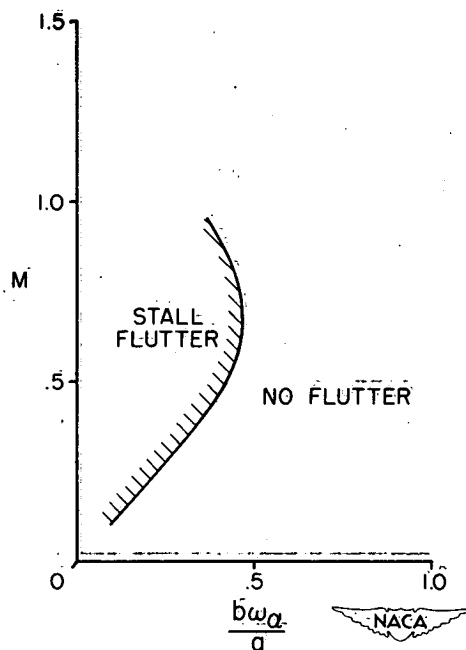


Figure 6.- Trend study of stall flutter.

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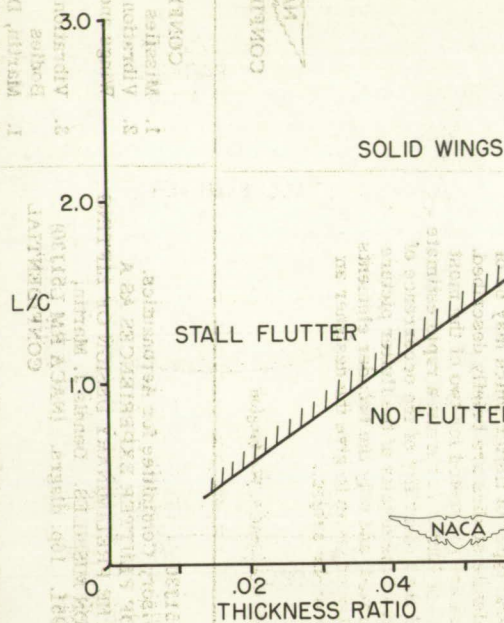


Figure 7.- Composite chart for stall flutter of solid unswept wings.

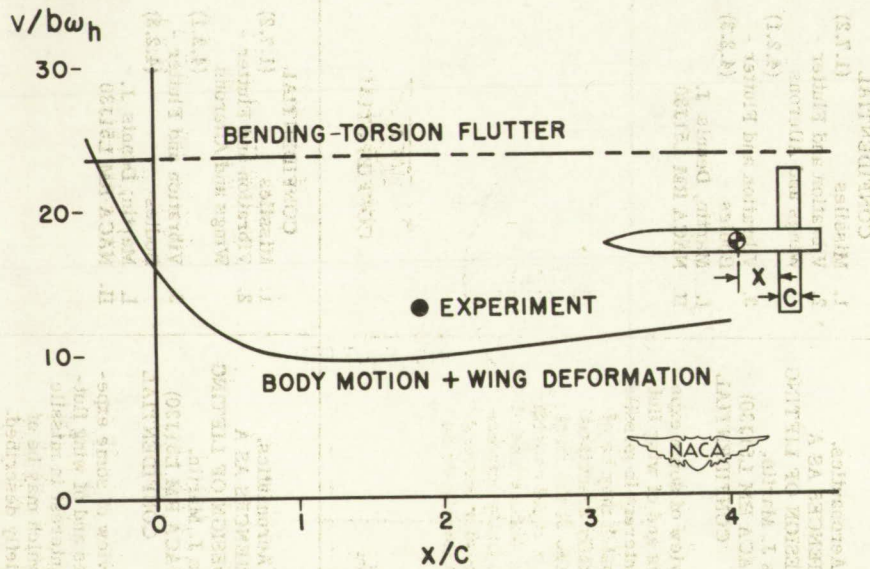


Figure 8.- Study of flutter involving body modes.

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